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13.1
Introduction

Scaling-up rules offer the potential to transfer knowledge obtained on small-scale laboratory equipment to large-scale production units. The principle of scaling-up is that equations describing the behavior of process equipment can be written in dimensionless form. If the resulting dimensionless groups are kept equal in the small-scale and in the large-scale equipment, the solutions of the various equations remain constant in dimensionless form.

Scaling-up of thermoplastic starch extrusion suffers from the same general problems that are encountered in many other processes in the process industry:

- on scaling up, the surface-to-volume ratio decreases and therefore the possibilities for heat transfer are limited in large-scale equipment,
- at equal temperature, differences in the temperature gradients, and therefore the heat fluxes, are smaller in large-scale equipment, and
- at equal shear fields in large-scale and small-scale equipment, diffusion limitations associated with distributive mixing can be more predominant in large extruders.

There are various theories on the scaling-up of single screw extruders. Because of the high viscosities involved, a considerable proportion of the process energy is transformed into heat by viscous dissipation. The thermal considerations will therefore dominate the scaling-up rules, and an important aspect is the extent to which the process is adiabatic or not. If the process can be considered to occur adiabatically, a sufficient condition for scaling-up will be that the energy input per unit throughput is constant and the average temperature of the end product will be the same in the small-scale and in the large-scale equipment. If this is not the case, similar temperature profiles in both types of equipment – called complete thermal similarity – are required.

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The degree to which a process is adiabatic can be estimated from the Brinkmann number (*Br*), which can be rewritten for extruders as:

$$Br = \frac{\mu v^2}{\lambda \Delta T} = \frac{\mu(\pi ND)^2}{\lambda \Delta T} \tag{13.1}$$

where λ is the thermal conductivity of the starch mass ($\mathrm{W\,m^{-2}\,K^{-1}}$) and ΔT is the temperature difference between the mass and the barrel wall. If this Brinkmann number is much larger than unity, adiabatic scaling-up is acceptable.

A particular dependency is the quadratic occurrence of the diameter. This implies that the Brinkmann number is generally large for production machines. It is generally not possible to keep the Brinkmann number constant for large-scale and small-scale machines. To obtain reliable predictions on a small-scale machine the Brinkmann number for this machine should at least be much larger than unity, which set its limitations to the minimum screw diameter of the small scale machine. If this number is smaller than unity for laboratory machines, reliable scaling-up is not possible.

In order to obtain complete thermal similarity, the screw rotation rate has to be decreased drastically, relative to the adiabatic case, with increasing screw diameter. As a result, the scale factor for the throughput is only 1.5 for Newtonian fluids (and decreases even further for fluids with pseudo-plastic behavior). This scaling-up factor (q) for the throughput is defined from:

$$\left[\frac{Q}{Q_0}\right] = \left[\frac{D}{D_0}\right]^q \tag{13.2}$$

where Q denotes the throughput, D the screw diameter, and the subscript 0 indicates the small extruder. In the case of adiabatic scaling-up a scaling-up factor of up to 3 can be achieved. For standard industrial extruder series it may be stated to a first approximation that:

$$\left[\frac{Q}{Q_0}\right] = \left[\frac{D}{D_0}\right]^{2.8}$$

When an extruder is scaled it is important to keep the process in the large machine as similar as possible to that in the small machine. Complete similarity is often not possible or it is impractical, so choices in similarity have to be made. Several types of similarities can play a role in the scaling-up of an extruder:

- ***Geometric similarity*** exists if the ratio between any two length parameters in the large-scale equipment is the same as the ratio between the corresponding lengths in the small-scale model. This is not necessarily the case, as will be seen later, but in general this condition can be very convenient.
- ***For hydrodynamic similarity*** two requirements should be fulfilled: the dimensionless flow profiles should be equal and, for twin-screw extruders, both

extruders should have the same (dimensionless) filled length. Equal dimensionless flow profiles lead to equal shear rates in corresponding locations, but not to equal velocities.

- ***Similarity in residence times*** means equal residence times in the small-scale and the large-scale equipment. This is not a requirement that is often fulfilled in extrusion processes, and in thermoplastic starch extrusion this can only be achieved if the scaling-up is adiabatic.
- ***Absolute thermal similarity*** is difficult to achieve, as stated before. This similarity indicates equal temperatures in all corresponding locations. A distinction has to be made between processes with small heat effects and those with large heat effects. For adiabatic processes in which the heat generation is far more important than heat removal to the wall, similarity based on overall energy balances is generally used. Although, strictly speaking, this does not lead to thermal similarity, equal average end temperatures of the product lead to far more favorable scaling-up rules.

13.2 Basic Analysis

To derive rules for scaling-up, all parameters are assumed to be related to the diameter ratio by a power relation. For this purpose, in this chapter all basic parameters are written in capitals, whereas the scaling-up factors are written in small print. This implies that all relevant parameters can be related to the screw diameter according to:

$$\left[\frac{N}{N_0}\right]=\left[\frac{D}{D_0}\right]^n;\left[\frac{P}{P_0}\right]=\left[\frac{D}{D_0}\right]^p;\left[\frac{\mu}{\mu_0}\right]=\left[\frac{D}{D_0}\right]^\upsilon;$$
$$\left[\frac{H}{H_0}\right]=\left[\frac{D}{D_0}\right]^h;\ \left[\frac{L}{L_0}\right]=\left[\frac{D}{D_0}\right]^l;\left[\frac{\tau}{\tau_0}\right]=\left[\frac{D}{D_0}\right]^t;\left[\frac{R}{R_0}\right]=\left[\frac{D}{D_0}\right]^r \tag{13.3}$$

where N is the rotation rate of the screws, P is the die pressure, and μ is the viscosity. The back flow (Q_b) is an important parameter in scaling up. For closely intermeshing twin-screw extruders it signifies the total amount of leakage flows, whereas for single-screw, self-wiping, and non-intermeshing extruders it signifies the pressure flow. H denotes the cannel depth, L the screw length, τ the residence time in the extruder, and R the pumping efficiency

For thermal scaling-up rules, two more parameters have to be used: the Greaz number (Gz), defined later, and the Brinkmann number (Br). The scaling-up notation for these dimensionless groups reads:

$$\left[\frac{Gz}{Gz_0}\right]=\left[\frac{D}{D_0}\right]^{gz} \ and \left[\frac{Br}{Br_0}\right]=\left[\frac{D}{D_0}\right]^{br} \tag{13.4}$$

13.3
Summary of Equations Used

Scaling-up rules are necessarily rather mathematical in nature. In this paragraph the extruder equations used are summarized.

The throughput of a single screw extruder can be written as:

$$Q = \frac{1}{2}\pi^2 ND^2 H(1-a)\sin\theta\cos\theta \tag{13.5}$$

θ is the flight angle and a is the throttle coefficient:

$$a = \frac{H^2 \Delta P \tan\theta}{6\mu(\pi ND)L} \tag{13.6}$$

The equation for the motor power in the pump zone can be written as:

$$E = \frac{(\pi ND)^2 WL}{H\sin\theta}(\cos^2\theta + 4\sin^2\theta + 3a\cos^2\theta) \tag{13.7}$$

where W is a channel width.

For use in scaling rules this equation can be simplified for screws with the same flight angle to:

$$E = const * \frac{\mu D^3 N^2 L}{H} \tag{13.8}$$

The pumping efficiency of the extruder is the ratio of energy used for pumping the material and the total energy input into the extruder.

$$R = \frac{QP}{E} \tag{13.9}$$

Thermal similarity yields from the energy balances:

$$\rho C_p\left(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z}\right) = \lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + q \tag{13.10}$$

In this equation q is the heat produced by viscous dissipation:

$$\begin{aligned} q = {} & 2\mu\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right\} + \\ & \mu\left\{\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)^2 + \left(\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right)^2\right\} \end{aligned} \tag{13.11}$$

If the equations above are made dimensionless there remain two important dimensionless numbers that govern the heat balances in the extruder: the Graez number and the Brinkmann number.

$$Gz = \frac{UH^2}{aL} \quad \text{and} \quad Br = \frac{\mu U^2}{\lambda \Delta T} \tag{13.12}$$

where λ is the thermal conductivity, T is the temperature, and $U = \pi ND$.

The Graez number accounts for the development of the temperature profile, whereas the Brinkmann number signifies the ratio between viscous dissipation and heat conduction to the wall.

13.4 Kinematic Similarity

Kinematic similarity means equal shear levels in the small and the large extruder. Its importance is coupled to the requirements for:

- equal mixing in small and large machines,
- equal distribution of viscous dissipation, and
- equal influence of non-Newtonian rheological effects.

For the throughput of the small laboratory extruder we can write:

$$Q_0 = \frac{1}{2}\pi^2 N_0 D_0^2 H_0 (1 - a_0) \sin\theta_0 \cos\theta_0 \tag{13.13}$$

and for the throughput of the production machine:

$$Q = \frac{1}{2}\pi^2 N D^2 H (1 - a) \sin\theta \cos\theta \tag{13.14}$$

If the screws of small and large machine have the same screw angle, which is the same as the same dimensionless pitch, we may write:

$$\frac{Q}{Q_0} = \frac{ND^2H}{N_0 D_0^2 H_0} \frac{(1-a)}{(1-a_0)} \tag{13.15}$$

and if we process both machines with the same throttle coefficient:

$$\frac{Q}{Q_0} = \frac{N}{N_0}\left(\frac{D}{D_0}\right)^2 \frac{H}{H_0} \tag{13.16}$$

Introduction of the diameter ratios as defined before:

$$\left(\frac{D}{D_0}\right)^q = \left(\frac{D}{D_0}\right)^n \left(\frac{D}{D_0}\right)^2 \left(\frac{D}{D_0}\right)^h = \left(\frac{D}{D_0}\right)^{n+2+h} \tag{13.17}$$

gives the exponent equation:

$$q = n + 2 + h \tag{13.18}$$

Because both machines operate with the same throttle coefficient:

$$a = a_0 \rightarrow \frac{H^2 \Delta P}{6\mu(\pi ND)L} \tan\theta = \frac{H_0^2 \Delta P_0}{6\mu_0(\pi N_0 D_0)L_0} \tan\theta_0 \tag{13.19}$$

and equal throttle coefficients lead to:

$$2h + p - \upsilon - 1 - n - l = 0 \tag{13.20}$$

For equal velocity gradients an extra equation is necessary:

$$\frac{\pi ND}{H} = \text{constant}$$

and therefore:

$$h = n + 1 \tag{13.21}$$

For kinematic similarity both (13.19) and (13.20) must be valid:

$$p = \ell - h + \upsilon \tag{13.22}$$

These results have to be combined with geometrical considerations of thermal scaling rules.

13.5 Geometrical Similarity

Geometrical similarity is often used for its simplicity, but it is not a strong requirement. Especially in processing thermoplastic starches, in which temperature and temperature homogeneity are very important, the principle of geometric similarity of small- and large-scale equipment cannot always be retained. Geometric similarity means that all dimensions scale in the same way, or:

$$l = 1 \text{ and } h = 1 \tag{13.23}$$

Geometric and kinematic similarity follows from a combination of this equation with Equations 13.1, 13.3 and 13.4, resulting in

$$n = 0; q = 3 \text{ and } p = \upsilon \tag{13.24}$$

This means for our process that:

- rotation speed must remain the same,
- throughput is proportional to D^3, and
- the die should be designed such that the pressure ratio equals the ratio between the end viscosities.

13.6 Motor Power and Torque

The motor power in the extruder can be approximated to:

$$E = const * \frac{\mu D^3 N^2 L}{H} \tag{13.25}$$

It should be realized that this equation does not include the power needed to transport the solid bed; however, this last item is not important for the thermal considerations in the next paragraphs.

The scale factor of the motor power can be defined as:

$$\frac{E}{E_0} = \left(\frac{D}{D_0}\right)^e \tag{13.26}$$

and we find:

$$e = 3 + 2n + \ell + \upsilon - h \tag{13.27}$$

and for the torque:

$$m = 3 + n + \ell + \upsilon - h \tag{13.28}$$

13.7 Equal Average End Temperature

Two types of thermal similarities can be used: equal average end temperatures and similar temperature profiles. The concept of equal average end temperatures can be applied if the extruder operates adiabatically or if $Br >> 1$. In this case scaling-up has to proceed according to equal motor power per unit throughput:

$$\frac{E}{Q} = const \quad or \quad e - q = 0 \tag{13.29}$$

With Equations 13.1 and 13.5 this leads for equal viscosities ($\psi = 0$) to

$$2h = 1 + n + l \tag{13.30}$$

In this case various degrees of freedom are still retained.

13.8
Similar Temperature Profiles

From the dimensionless energy equation it follows that thermal similarity can be attained if the dimensionless numbers of Graez and Brinkmann are the same for both sizes of machines.

Because:

$$Br = \frac{\mu(\pi ND)^2}{\lambda \Delta T} \tag{13.31}$$

we find for materials with the same heat conductivity (λ) that thermal similarity is attained if:

$$\upsilon + 2n + 2 = 0 \tag{13.32}$$

For materials with the same viscosity, this means: $n = -1$.

From:

$$Gz = \frac{\pi NDH^2}{aL} \tag{13.33}$$

it follows at equal heat diffusivity that:

$$1 + n + 2h - \ell = 0 \tag{13.34}$$

leading to thermal similarity (equal Br and Gz numbers) if:

$$2h = l \tag{13.35}$$

For extruders with equal length-to-diameter ratios ($\ell = 1$) this means that the channel depth must decrease according to $h = ½$, which together with Equation 13.18 gives:

$$q = 2 + n + h = 1.5 \tag{13.36}$$

or:

$$\frac{Q}{Q_0} = \left[\frac{D}{D_0}\right]^{1.5} \tag{13.37}$$

From the economical point of view this is very unfavorable, and should only be applied in very special situations.

13.9
Similarity in Residence Times

Equal residence time can be achieved if the volume divided by the throughput remains constant, or, if we define Z as the average residence time:

$$Z = const \frac{HLW}{Q} \tag{13.38}$$

which for screws with equal helix angle gives:

$$z = h + 1 + l - q \tag{13.39}$$

or with:

$$q = 2 + n + h \tag{13.40}$$

we find that:

$$z = l - n - 1 \tag{13.41}$$

For screws with geometric similarity – this means that ($l = -1$, $h = 1$ and $z = -n$) – equal residence times are only possible if the rotation speed is constant. In other cases equal residence times can only be obtained by changing the screw length, according to:

$$l = 1 + n \tag{13.42}$$

13.10
Guidelines for Scaling

In extrusion of thermoplastic starches, both heat of conduction and heat of dissipation are generally important in the process. In small machines the Brinkmann number is relatively small, but in larger machines the dissipation becomes more dominant and the process becomes more adiabatic. Because the thermal problems

Table 13.1 Application of Equation 13.30, giving a variety of possibilities for scaling rules.

n	*h*	*q*
–1.0	0.5	1.5
–0.6	0.7	2.1
–0.4	0.8	2.4
0.0	1.0	3.0

are predominant, the basis for the guide lines is Equation 13.30, which can be combined with various other (less strict) requirements. Application of Equation 13.30 gives a variety of possibilities for scaling rules, which give for screws with equal length to diameter ratio, for instance (Table 13.1):

Equal end temperatures with adiabatic operation still leave the degrees of freedom to scale according to similar temperature profiles (of course!) with $q = 1.5$ or to scale kinematically with $q = 3$ and with values in between. For the design of extruders for thermoplastic starches this means that the thermal stability of the material and of the process are important. For the compounding process for the preparation of the starch it can be envisioned that kinematic scaling-up should be preferred because temperature effects are still mildly important but kinematic similarity is important to achieve the same mixing mechanism (and therefore the same material) in the small- and the large-scale processes. For processes such as film blowing, on the other hand, thermal similarity is extremely important, leading to thermal scaling-up. Profile extrusion and sheet extrusion are "in-between" processes and could be designed with $n = -0.4$ and $h = 0.8$, leading to $q = 2.4$.

In the examples above the L/D ratio remains constant but the screw length can also be changed to retain extra degrees of freedom. This leads to a three-dimensional matrix of parameters, but that is outside the scope of this work.

References

1 Brouwer, T., Todd, D.B. and Janssen, L.P.B.M. (1998) Drag and pressure flow with special twin screw mixing elements. Proc. Polymer Proc. Soc., North American Meeting, Toronto CDN, August, 17–19, 1998, pp. 30–1.

2 Bruin, S., van Zuilichem, D.J. and Stolp, W. (1978) A review of fundamental and engineering aspects of extrusion of biopolymers in single-screw extruder. *Journal of Food Process Engineering*, **2**, 4–17.

3 Goffard, D., van der Wa1, D.J., Klomp, E.M., Hoogstraten, H.W., Janssen, L.P.B.M., Breysse, L. and Trolez, Y. (1996) Three-dimensional flow modelling of a self-wiping corotating twin screw-extruder, Part l. The Transporting Section. *Polymer Engineering and Science*, **36**, 901–11.

4 de Graaf, R.A., Rohde, M. and Janssen, L.P.B.M. (1997) A novel model predicting the residence time distribution during reactive extrusion. *Chemical Engineering Science*, **52**, 4345–56.

5 Harper, J.M. (1981) *Extrusion of Foods*, CRC Press Inc., Boca Ration, FL.

6 Janssen, L.P.B.M., Moscicki, L. and Mitrus, M. (2002) Energy balance in food extrusion-cooking. *International Agrophysics*, **16** (**3**), 191–5.

7 Janssen, L.P.B.M., Rozendal, P.F., Hoogstraten, H.W. and Cioffi, M. (2001) A dynamic model for multiple steady states in reactive extrusion. *International Polymer Processing*, **XVI**, 263–71.

8 Janssen, L.P.B.M. (1998) On the stability of reactive extrusion. *Polymer Engineering and Science*, **38**, 2010–19.

9 Tsao, T.F., Harper, J.M. and Repholz, K.M. (1978) The effects of screw geometry on extruder operational characteristics. *AIChE Symposium Series*, **7** (**172**), 142.

10 Wal, D.J., van der Goffard, D., Klomp, E.M., Hoogstraten, H.W. and Janssen, L.P.B.M. (1996) Three-dimensional flow modelling of a self-wiping corotating twin-screw extruder, Part II. The Kneading Section. *Polymer Engineering and Science*, **36**, 912–24.

11 Yacu, W.A. (1983) Modelling of a two-screw co-rotating extruder, in *Thermal Processing and Quality of Foods*, Elsevier Applied Science Publishers, London.

12 van Zuilichem, D.J. and Janssen, L.P.B.M. (1980) *Reology*, Vol. **3**, Plenum Press, New York and London.

13 van Zuilichem, D.J. (1992) Extrusion cooking. Craft or science? PhD thesis. Wageningen University, the Netherlands.